KSUPT-03/2 February 2003

Median Statistics and the Mass Density of the Universe

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ABSTRACT

We use weighted mean and median statistics techniques to combine individual estimates of $\Omega_{\rm M0}$, the present mean mass density in non-relativistic matter, and determine the observed values and ranges of $\Omega_{\rm M0}$ from different combinations of data. The derived weighted mean $\Omega_{\rm M0}$ values are not good representatives of the individual measurements, under the assumptions of Gaussianity and negligible correlation between the individual measurements. This could mean that some observational error bars are under-estimated. Discarding the most discrepant ~ 5 % of the measurements generally alleviates but does not completely resolve this problem. While the results derived from the different combinations of data are not identical, they are mostly consistent, and a reasonable summary of the median statistics analyses is $0.2 \lesssim \Omega_{\rm M0} \lesssim 0.35$ at two standard deviations.

Subject headings: cosmology: cosmological parameters—cosmology: observation—methods: statistics—methods: data analysis—large-scale structure of the universe

1. Introduction

Oftentimes it is useful to combine results from many different measurements of a quantity and derive a more accurate estimate of that quantity. Thus Gott et al. (2001) study a collection of all available pre-mid-1999 estimates of the present value of Hubble's constant, H_0, and derive

$$H_0 = 100h\,{\rm km\,s^{-1}Mpc^{-1}} = 67 \pm 7\,{\rm km\,s^{-1}Mpc^{-1}}, \tag{1}$$

at two standard deviations[1], a significantly more constraining estimate of H_0 than is provided by any single measurement.

Similar meta-analysis techniques have been used to determine binned multipole-space cosmic microwave background (CMB) anisotropy power spectra by combining many different CMB anisotropy measurements (see, e.g., Podariu et al. 2001; Miller et al. 2002; Page 2002; Wang et

[1] Where the first equation defines the dimensionless parameter h and we halve the 2 σ error bar of Gott et al. (2001) to get a 1 σ error bar for our computations.

al. 2002; Mukherjee & Wang 2003), and to derive constraints on cosmological-model-parameters from combined CMB anisotropy data sets (see, e.g., Ödman et al. 2002; Mukherjee et al. 2002; Douspis et al. 2002; Wang et al. 2002).

The more widely used weighted mean technique, discussed in Podariu et al. (2001), assumes Gaussian errors.[2] Thus in this case one may compute a goodness-of-fit parameter, and the number of standard deviations, N_σ, this parameter deviates from what is expected (Podariu et al. 2001). A large value of N_σ could indicate the presence of unaccounted for systematic uncertainties, the breakdown of the Gaussian assumption, or the presence of significant correlations between the individual measurements used.

The other technique we use, that based on median statistics (Gott et al. 2001)[3], does not assume that the measurement errors are Gaussian, or even that the magnitude of these errors are known. It assumes only that the measurements are independent and free of systematic errors. It is hence not possible to estimate the goodness of fit in the median statistics case. However, since the median statistics technique is based on fewer assumptions than the weighted mean technique, median statistics results are more robust, but still — as Gott et al. (2001) show — almost as constraining as weighted mean results.

In this paper we apply both these techniques to collections of estimates of $\Omega_{\rm M0}$, the present value of the mean mass density of non-relativistic matter in the universe. A robust and tight estimate of $\Omega_{\rm M0}$ is of great interest. Current indications are that $\Omega_{\rm M0}$ is small and we live in a low-density universe (see Peebles & Ratra 2003 for a review). This, in conjunction with recent CMB anisotropy measurements which suggest that the curvature of spatial hypersurfaces is small, indicates that a dark energy dominated spatially-flat universe (see, e.g., Peebles 1984; Peebles & Ratra 1988, 2003; Steinhardt 1999; Sahni & Starobinsky 2000; Carroll 2001; Padmanabhan 2002) is observationally favored over a spatially open model with insignificant dark energy density (see, e.g., Gott 1982; Ratra & Peebles 1995). To strengthen this conclusion it would be helpful to have in hand a more robust and tight estimate of $\Omega_{\rm M0}$ than is available from any single measurement.[4]

[2]A number of quantities of interest, e.g., the CMB anisotropy spectrum, are commonly thought to have been generated by quantum fluctuations in a weakly coupled field during an early epoch of inflation and are thus realizations of spatially stationary Gaussian random processes (see, e.g., Ratra 1985; Fischler, Ratra, & Susskind 1985). Measurements appear to be consistent with this Gaussianity assumption — for discussions of the Gaussianity of the smaller-scale CMB anisotropy see, e.g., Park et al. (2001), Shandarin et al. (2002), Santos et al. (2002), and Polenta et al. (2002) — and so in cases where the experimental noise is Gaussian it is fair to use the weighted mean technique.

[3]See Avelino, Martins, & Pinto (2002) for a recent application of the Gott et al. (2001) median statistics technique.

[4]Of course, comparing the predictions of dark energy dominated models to observational measurements is another way to check for the presence of dark energy. In the near future neoclassical cosmological tests that hold significant promise include those based on CMB anisotropy (see, e.g., Brax, Martin, & Riazuelo 2000; Amendola et al. 2002), gravitational lensing (see, e.g., Ratra & Quillen 1992; Waga & Frieman 2000; Chae et al. 2002), Type Ia supernova redshift-apparent magnitude (see, e.g., Podariu & Ratra 2000; Waga & Frieman 2000; Leibundgut 2001), redshift-counts (see, e.g., Huterer & Turner 2001; Podariu & Ratra 2001; Levine, Schulz, & White 2002), and redshift-angular

While it would be useful to focus on measurements of $\Omega_{\rm M0}$ that are independent of cosmological model, we have been able to locate only 30 such recent smaller-scale estimates of $\Omega_{\rm M0}$. To reduce statistical uncertainty it is desirable to have a greater number of $\Omega_{\rm M0}$ estimates. We hence also consider recent $\Omega_{\rm M0}$ estimates derived assuming either a spatially-flat model with a cosmological constant Λ or an open model with no Λ.

The $\Omega_{\rm M0}$ measurements we focus on are listed and discussed in § 2. Results are presented and discussed in § 3. We conclude in § 4.

2. $\Omega_{\rm M0}$ Measurements

Tables 1—3 list the values and errors bars of $\Omega_{\rm M0}$ for the measurements we consider. Table 1 lists values determined in a manner that is independent of cosmological model, while Tables 2 and 3 list values derived assuming a spatially flat Λ dominated model, and a spatially open model with no Λ, respectively. In general, in these Tables, we include quoted systematic errors in quadrature when determining the total error bar and assume a Gaussian distribution when determining 1 σ errors (if these are not given). In what follows we briefly describe how we determine the $\Omega_{\rm M0}$ values and error bars given in these Tables.

2.1. Redshift Distortion Factor

There are many measurements of the redshift distortion factor $\beta = \Omega_{\rm M0}^{0.6}/b$, where $\Omega_{\rm M0}^{0.6}$ is a reasonably accurate approximation of the velocity function evaluated at zero redshift $f(z = 0)$ (Peebles 1993, § 13) and b is the bias factor for the tracer used, defined in terms of the ratio of the fractional density perturbations, $b = \delta_{\rm trace}/\delta_{\rm mass}$, where $\delta_{\rm trace}$ is the fractional number density perturbation.

To determine $\Omega_{\rm M0}$ from β we need to know the bias factor. In this paper, we use, for optical galaxies (Verde et al. 2002; Lahav et al. 2002; Peacock et al. 2002)[5]

$$b_O = 1.0 \pm 0.1, \tag{2}$$

and for infrared galaxies and clusters of galaxies (Plionis et al. 2000),

$$b_I = b_O/(1.21 \pm 0.06), \qquad b_C = b_I(4.3 \pm 0.8), \tag{3}$$

all at one standard deviation.

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size (see, e.g., Zhu & Fujimoto 2002; Chen & Ratra 2003; Podariu et al. 2003) data.

[5]We average the values given in these papers to get the b_O value quoted here.

Typically, β is measured from density-density comparisons (D-D in the Tables) or velocity-velocity comparisons (V-V in the Tables), or through the distortion effect of peculiar velocities on redshift surveys. By using the somewhat related least-action principle, Susperregi (2001) is able to independently determine $\Omega_{\rm M0}$ and b, and we quote his value of $\Omega_{\rm M0}$ in Table 1.

2.2. Power Spectrum

A commonly used simple analytic fit to the observed power spectrum of cosmological mass fluctuations is the CDM spectrum (see, e.g., Peacock 1999, § 16.8). In this approximation, the shape and amplitude of the mass power spectrum depends on two parameters: the shape parameter Γ, and σ_8, the rms fractional mass density variation averaged over $8h^{-1}$ Mpc spheres. Nowadays, the shape parameter is usually approximated by (Sugiyama 1995),

$$\Gamma = \Omega_{\rm M0} h e^{-\Omega_B (1+\sqrt{2h}/\Omega_{\rm M0})} \tag{4}$$

where Ω_B is a measure of the present mean mass density in baryonic matter. Occasionally however the shape parameter is still defined through $\Gamma = \Omega_{\rm M0} h$.

To extract a value and error bars for $\Omega_{\rm M0}$ from a measurement of Γ we need the value of h — we use eq. (1) for this — and an estimate of the baryonic mass density parameter Ω_B, for which we use

$$\Omega_B h^2 = 0.014 \pm 0.004, \tag{5}$$

at 1 σ, derived by averaging the two extreme values quoted in § IV.B.2 of Peebles & Ratra (2003).

2.3. Velocity Correlation

By assuming a shape for the power spectrum, the velocity correlation method provides a constraint on a function of σ_8 and $\Omega_{\rm M0}^{0.6}$ by comparing the observed velocity correlation to that predicted from the power spectrum. Given an estimate of σ_8, for which we use[6]

$$\sigma_8 = 0.94 \pm 0.11, \tag{6}$$

at 1 σ, we can determine $\Omega_{\rm M0}$. Juszkiewicz et al. (2000) consider a variant of this method based on relative velocities of galaxies.

[6]Here the rms factional mass density variation averaged over $8h^{-1}$ Mpc spheres $\sigma_8 = \sigma_{8\rm trace}/b_{\rm trace}$, where $\sigma_{8\rm trace}$ is the corresponding rms number density variation and $b_{\rm trace}$ is the bias factor for the tracer used. We average the values given for σ_8 in eq. (37) of Hamilton & Tegmark (2002) and eq. (12) of Szalay et al. (2001).

2.4. Gas Mass Fraction

Assuming the baryonic mass fraction in galaxy clusters is an accurate representation of that of the universe, and given Ω_B, the baryon mass fraction f_b provides an estimate of $\Omega_{\rm M0}$,

$$\Omega_{\rm M0} = \Omega_B / f_b. \tag{7}$$

The related gas mass fraction of galaxy clusters, f_g, is what is measured, and it provides the estimate

$$\Omega_{\rm M0} = \frac{\Omega_B}{f_g(1 + 0.19h^{0.5})}, \tag{8}$$

where we use h and Ω_B from eqs. (1) and (5).

2.5. Mass to Light Ratios

This method assumes that the mass-to-light ratios of galaxy clusters are accurate representatives of that of the whole universe. Table 1 lists the Carlberg et al. (1997a) and Bahcall et al. (2000) $\Omega_{\rm M0}$ values and 1 σ ranges. We also show results derived from the Hradecky et al. (2000) data; to determine the $\Omega_{\rm M0}$ central value and 1 σ range, we compute the weighted mean and error bar of M/L_V using Table 5 of their paper and use the Efstathiou, Ellis, & Peterson (1988) estimate $\langle L \rangle \approx (2 \pm 0.7) \times 10^8 h L_\odot$ Mpc^{-3}.

2.6. Cosmological-Model-Dependent Estimates

Tables 2 and 3 list $\Omega_{\rm M0}$ values determined assuming a flat-Λ and an open cosmological model, respectively. Such model-dependent estimates are becoming more common. Weak lensing (WL) measurements have recently begun to provide interesting constraints on a function of $\Omega_{\rm M0}$ and σ_8, while improving galaxy cluster number density measurements (both at the present epoch and as a function of redshift) constrain a related function. Using the estimate for σ_8 given in eq. (6) we may use these constraints as measurements of $\Omega_{\rm M0}$.

Table 2 also lists estimates of $\Omega_{\rm M0}$ from various other methods, including cosmic microwave background (CMB) anisotropy measurements, the angular size versus redshift ($\theta - z$) test, strong gravitational lensing, and the supernova apparent magnitude versus redshift test.

3. Methods and Results

Table 1 lists 30 "model-independent" measurements of $\Omega_{\rm M0}$, while Tables 2 and 3 show 28 and 14 results determined assuming a flat-Λ and an open model, respectively. In addition to these three data sets, we also consider two additional combination data sets: combinations of the 30 model-independent results with the flat-Λ model values and with the open model values.

For each of these five data sets, we compute the weighted mean of $\Omega_{\rm M0}$ and the associated error estimate as follows (see, e.g., Podariu et al. 2001). The standard expression for the weighted mean is

$$\Omega_{\rm M0} = \frac{\sum_{i=1}^{N}(\Omega_{\rm M0})_i/\sigma_i^2}{\sum_{i=1}^{N} 1/{\sigma_i}^2}, \tag{9}$$

where $i = 1, 2, \ldots N$ indexes the N measurements in the data set, with central values $(\Omega_{\rm M0})_i$ and errors σ_i. The (internal) error estimate for each data set is

$$\sigma = \left(\sum_{i=1}^{N} 1/{\sigma_i}^2\right)^{-1/2}. \tag{10}$$

The goodness of fit parameter is

$$\chi^2 = \frac{1}{N-1}\sum_{i=1}^{N}\frac{\left((\Omega_{\rm M0})_i - \Omega_{\rm M0}\right)^2}{{\sigma_i}^2} = \sum_{i=1}^{N}\chi_i^2, \tag{11}$$

where the last equation defines χ_i^2, the "reduced χ^2" contribution from each measurement. Since the weighted mean technique assumes Gaussian errors, χ has expected value unity with error $1/\sqrt{2(N-1)}$, so the number of standard deviations that χ deviates from unity is

$$N_\sigma = |\chi - 1|\sqrt{2(N-1)}. \tag{12}$$

A large value of N_σ could indicate the presence of unaccounted for systematic errors, the invalidity of the Gaussian assumption, or the presence of significant correlations between the measurements.

We also analyze each of the five data sets using median statistics (see, e.g., Gott et al. 2001; Podariu et al. 2001). For each data set, we construct the distribution for the true median $\Omega_{\rm M0}$ value using the binomial theorem method of eq. (1) of Gott et al. (2001).[7] Since $\Omega_{\rm M0}$ is positive, following Gott et al. (2001), we integrate over this distribution with a logarithmic prior between data points to determine confidence intervals for $\Omega_{\rm M0}$.[8]

Table 4 shows the results for the weighted mean and median statistics analyses. The upper half of the table shows results derived using all measurements. The weighted mean technique results in tighter constraints on $\Omega_{\rm M0}$, while the median statistics constraints are weaker. This result (Gott

[7]For example, if there were two measurements of a given quantity, x_1 and x_2 ($x_1 < x_2$), then there is a 25 % probability that the true median lies below x_1, a 50 % probability that it lies between x_1 and x_2, and a 25 % probability that it lies above x_2.

[8]Lower and upper confidence levels at 1 and 2 σ significance are determined such that the probabilities inside the 1 and 2 σ ranges are divided in half by the median value.

et al. 2001; Podariu et al. 2001) is reinforced by the large N_σ values in the upper half of Table 4. For Gaussian distributed errors, N_σ is a measure of how well the weighted mean and derived error bar represent the measurements considered. N_σ is greater than 2 in all cases, i.e., χ is more than 2 σ away from what is expected for Gaussian distributed errors. This most likely means that one (or more) of the measurements has an underestimated error bar.[9] Since median statistics do not make use of the measurement error bars, the median statistics results are likely more reliable than the weighted mean results.

To examine the issue of large N_σ values, we proceed as follows. For each measurement in each of the five data sets, we compute χ_i^2, eq. (11). We then discard the ~ 5 % most discrepant (largest χ_i^2) measurements from each data set and so generate five culled data sets of "good" measurements. For the model-independent data set, we drop only one measurement, that from the least-action principle method (Susperregi 2001), which has a small error bar and $\chi_i^2 = 0.99$. For the flat-Λ data set we have to drop two measurements: the Zaroubi et al. (1997) v-correlation result which has a large $\Omega_{\rm M0}$ and $\chi_i^2 = 0.29$, and the Allen et al. (2002b) cluster number density measurement which has a small $\Omega_{\rm M0}$ and $\chi_i^2 = 0.27$. For the open model data set we drop the Hamana et al. (2002) weak lensing measurement which has a small $\Omega_{\rm M0}$ and $\chi_i^2 = 1.3$. These measurements are also the most discrepant ones in the two combination data sets, so we drop them again when generating the culled combination data sets. Of course, not all large N_σ values are reduced to unity by this culling (although they can be by further culling): we are only discarding the most "discrepant" measurements to investigate the stability (robustness) of the constraints on $\Omega_{\rm M0}$.

Results from the analyses of the culled data sets are shown in the lower half of Table 4. For the model-independent, flat-Λ, and their combination data sets, the weighted mean and median statistics error bars are in better accord, and N_σ are of order unity. For the open model data sets N_σ are smaller now, but still significantly large than unity, indicating perhaps that the error bars on one or more of the remaining measurements are underestimated.

Focussing on the median statistics 2 σ ranges for the culled data sets (the lower half of the last column of Table 4), if we exclude the open model results for the reason mentioned above, a reasonable summary is

$$0.2 \lesssim \Omega_{\rm M0} \lesssim 0.35 \tag{13}$$

at two standard deviations, with central value at $\Omega_{\rm M0} \sim 0.25 - 0.3$. It is reassuring that these summary values are in agreement with other recent estimates (see, e.g., Peebles & Ratra 2003).

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[9]Given the available evidence, it is reasonable to assume that the Gaussianity assumption is not invalid. In addition, we have attempted to select only those measurements that are not strongly correlated. Thus the large N_σ values we find are most likely a consequence of one (or more) measurements that have underestimated error bars.

4. Conclusion

We have determined a preliminary estimate of the mean mass density in nonrelativistic matter, from median statistics analyses of various collections of measurements. The results of our meta-analysis estimate of Ω_{M0} appear very reasonable. More high quality data, especially "model-independent" data, should allow for significantly more constraining limits on Ω_{M0}.

We acknowledge helpful discussions with R. Gott, J. Peebles, S. Podariu, and U. Seljak, and support from NSF CAREER grant AST-9875031.

Table 1. Ω_{M0} Values that are Independent of Cosmological Model

Method	Data Set	Ω_{M0}	Ω_{M0} (1 σ range)	Reference
z-distortion	2dFGRS	0.24	0.16–0.32	Peacock et al. (2002)
z-distortion	*IRAS*PSCz	0.15	0.07–0.23	Taylor et al. (2000)
V-V	ORS/SBF	0.13	0.08–0.18	Blakeslee et al. (2000)
V-V	*IRAS*/SBF	0.19	0.12–0.26	Blakeslee et al. (2000)
V-V	SFI/*IRAS*PSCz	0.17	0.13–0.21	Branchini et al. (2001)
V-V	MarkIII/*IRAS*	0.23	0.17–0.29	Willick & Strauss (1998)
V-V	ENEAR/*IRAS*PSCz	0.23	0.14–0.22	Nusser et al. (2001)
D-D	MarkIII/Optical	0.60	0.33–0.87	Hudson et al. (1995)
D-D	MarkIII/*IRAS*1.2Jy	0.60	0.42–0.78	Sigad et al. (1998)
D-D/V-V	Abell,ACO/MarkIII	0.66	0.20–1.1	Branchini et al. (2000)
dipole	XBACs	0.76	0.37–1.2	Plionis & Kolokotronis (1998)
LAP	PSCz,ORS,MarkIII,SFI	0.37	0.36–0.38	Susperregi (2001)
Γ	SDSS	0.33	0.26–0.40	Szalay et al. (2001)
Γ	REFLEX	0.34	0.25–0.42	Schuecker et al. (2001)
Γ	2dFQSO	0.20	0.04–0.36	Hoyle et al. (2002)
Γ	APM	0.25	0.11–0.39	Efstathiou & Moody (2001)
Γ	2dFGRS	0.30	0.25–0.35	Percival et al. (2001)
Γ	LCRS	0.24	0.09–0.39	Matsubara et al. (2000)
v-correlation	ENEAR	0.36	0.24–0.48	Borgani et al. (2000)
v-correlation	MarkIII	0.35	0.18–0.52[a]	Juszkiewicz et al. (2000)
f_g	SZ data	0.20	0.14–0.26	Grego et al. (2001)
f_g	RXJ2228+2037	0.12	0.07–0.16	Pointecouteau et al. (2002)
f_g	*ROSAT*PSPC,*ASCA*	0.25	0.16–0.33	Ettori & Fabian (1999)
f_g	*Chandra*	0.22	0.15–0.29	Allen et al. (2002a)
f_g	*BeppoSAX*,*Chandra*	0.28[b]	0.20–0.36[b]	Ettori et al. (2002)
f_b	*ROSAT*PSPC	0.44[c]	0.29–0.59[c]	Sadat & Blanchard (2001)
f_b	*ROSAT*,*Ginga*,*ASCA*	0.33	0.23–0.42	Roussel et al. (2000)
M/L		0.19	0.12–0.26	Carlberg et al. (1997a)
M/L		0.16	0.10–0.22	Bahcall et al. (2000)
M/L		0.15[d]	0.094–0.21[d]	Hradecky et al. (2000)

[a]From their Figure 2, by allowing σ_8 to vary between 0.83 and 1.05, as given in our eq. (6).

[b]From the weighted mean and the external error of f_{gas} (Δ=500) values in their Table 1.

[c]The 1 σ range of f_b is from the first rows of their Tables 4 and 6, and the central value is the mean of these 1 σ values.

[d]See the discussion in § 2.5.

Table 2. $\Omega_{\rm M0}$ Values Determined Assuming a Flat-Λ Cosmological Model

Method	Data Set	$\Omega_{\rm M0}$	$\Omega_{\rm M0}$ (1 σ range)	Reference
v-correlation	MarkIII	0.90	0.66–1.1	Zaroubi et al. (1997)
WL	VIRMOS-DESCART	0.30	0.21–0.39	Van Waerbeke et al. (2002)
WL	Keck,WHT	0.31	0.23–0.39	Bacon et al. (2002)
WL	COMBO-17	0.48	0.31–0.65	Brown et al. (2002)
WL	Suprime-Cam	0.15	0.06–0.24	Hamana et al. (2002)
WL	RCS	0.26	0.18–0.34	Hoekstra et al. (2002)
WL	CTIO	0.18	0.13–0.23	Jarvis et al. (2002)
WL	MDS	0.30	0.17–0.43	Refregier et al. (2002)
cluster		0.33	0.22–0.44	Viana & Liddle (1999)
cluster		0.09	0.04–0.14	Allen et al. (2002b)
cluster		0.44	0.32–0.56	Henry (2000)
cluster	REFLEX	0.34	0.25–0.44	Schuecker et al. (2002)
cluster		0.18	0.045–0.32	Seljak (2002)
cluster	ROSAT,ASCA	0.34	0.26–0.42	Pierpaoli et al. (2001)
cluster	EMSS,RASS	0.27	0.13–0.41	Donahue & Voit (1999)
cluster	HIFLUGCS	0.12	0.08–0.16	Reiprich & Böhringer (2002)
cluster	SDSS	0.18	0.13–0.23	Bahcall et al. (2003)
cluster	RDCS	0.35	0.23–0.47	Borgani et al. (2001)
cluster	CNOC	0.40	0.27–0.53	Carlberg et al. (1997b)
cluster		0.57	0.23–0.91	Bahcall & Fan (1998)
cluster		0.26	0.17–0.35	Wu (2001)
cluster		0.87	0.57–1.2	Blanchard et al. (2000)
cluster	HIFLUGCS,*Chandra*	0.26	0.14–0.38	Vikhlinin et al. (2002)
CMB anisotropy		0.38	0.20–0.56	Percival et al. (2002)
θ-z	radio galaxies	0.10	0.00–0.35	Guerra et al. (2000)
power spectrum	Lyα forest	0.25[a]	0.00–0.71[a]	Croft et al. (2002)
strong lensing	CLASS	0.31	0.08–0.54	Chae et al. (2002)
magnitude-z	supernova	0.28	0.18–0.38	Perlmutter et al. (1999)

[a]From their eq. (25), assuming the spectral index n=1 and using the values for Ω_B and h given in our § 2.

Table 3. Ω_{M0} Values Determined Assuming an Open Cosmological Model

Method	Data Set	Ω_{M0}	Ω_{M0} (1 σ range)	Reference
v-correlation	MarkIII	0.90	0.66–1.1	Zaroubi et al. (1997)
WL	VIRMOS-DESCART	0.27	0.21–0.33	Van Waerbeke et al. (2002)
WL	Suprime-Cam	0.04	0.00–0.08	Hamana et al. (2002)
WL	RCS	0.26	0.18–0.34	Hoekstra et al. (2002)
WL	FORS1	0.37	0.25–0.49	Maoli et al. (2001)
cluster		0.22	0.12–0.32	Viana & Liddle (1999)
cluster		0.49	0.37–0.61	Henry (2000)
cluster	EMSS,RASS	0.45	0.31–0.59	Donahue & Voit (1999)
cluster	HIFLUGCS	0.12	0.08–0.16	Reiprich & Böhringer (2002)
cluster	CNOC	0.40	0.27–0.53	Carlberg et al. (1997b)
cluster		0.51	0.14–0.88	Bahcall & Fan (1998)
cluster		0.18	0.08–0.28	Wu (2001)
cluster		0.92	0.69–1.2	Blanchard et al. (2000)
cluster	HIFLUGCS,*Chandra*	0.48	0.40–0.56	Vikhlinin et al. (2002)

Table 4. Weighted Mean and Median Statistics Results[a]

Data Set	N[b]	$\Omega_{\rm M0}^{\rm WM}$	$\Omega_{\rm M0}^{\rm WM}$(1 σ range)	$\Omega_{\rm M0}^{\rm WM}$(2 σ range)	N_σ[c]	$\Omega_{\rm M0}^{\rm MS}$	$\Omega_{\rm M0}^{\rm MS}$(1 σ range)	$\Omega_{\rm M0}^{\rm MS}$(2 σ range)
All Measurements								
Model-independent	30	0.32	0.31–0.32	0.30–0.33	7.9	0.24	0.23–0.28	0.20–0.33
Flat-Λ	28	0.22	0.21–0.24	0.19–0.26	2.4	0.30	0.27–0.31	0.26–0.34
Open	14	0.20	0.18–0.22	0.16–0.25	6.4	0.38	0.26–0.45	0.18–0.49
Flat-Λ & Model-ind.	58	0.30	0.29–0.30	0.28–0.31	9.0	0.28	0.25–0.30	0.24–0.32
Open & Model-ind.	44	0.30	0.29–0.31	0.29–0.32	11.	0.26	0.24–0.31	0.23–0.35
"Good" Measurements Only								
Model-independent	29	0.21	0.20–0.23	0.18–0.24	0.98	0.24	0.23–0.26	0.20–0.32
Flat-Λ	26	0.24	0.22–0.25	0.20–0.27	0.92	0.30	0.27–0.31	0.26–0.34
Open	13	0.26	0.24–0.29	0.21–0.31	4.4	0.40	0.27–0.46	0.22–0.50
Flat-Λ & Model-ind.	55	0.22	0.21–0.23	0.20–0.24	1.4	0.27	0.25–0.30	0.24–0.31
Open & Model-ind.	42	0.22	0.21–0.24	0.20–0.25	3.8	0.26	0.24–0.31	0.23–0.34

[a]Superscripts WM and MS indicate weighted mean and median statistics results, respectively.

[b]Number of measurements in the data set.

[c]Eq. (12).

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